Radiation Fields of Optical Stripline Waveguides

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Abstract—Dispersion characteristics and radiation fields of an optical stripline waveguide radiating into free space are calculated. The waveguides are fabricated as multiple layers of differing dielectric materials. A top layer is etched to form a "cap" with an effective waveguide in a layer below the cap. Confinement of the fields to the waveguide is obtained in the vertical direction by dielectric discontinuities, while lateral confinement occurs because of spatial interference of a continuum of plane waves. The radiation field of the fundamental mode in a plane perpendicular to the waveguide layers is characterized by the layer widths and index discontinuities. Beamwidths of the fundamental mode in the plane parallel to the dielectric layers are developed in terms of the waveguide parameters. Values of these parameters which yield the best optical confinement under the stripe can be obtained.

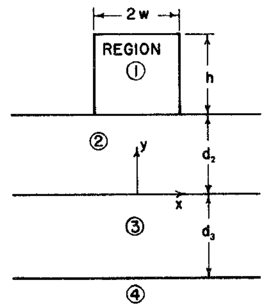

Fig. 1. Cross section of the optical stripline waveguide.

I. Introduction

THE OPTICAL stripline waveguide has potential applications as a low-loss channel waveguide in integrated optical circuits [1]. The interest in this waveguide is due to the fact that it is relatively simple to fabricate compared to buried waveguide structures having built-in dielectric steps in both transverse directions. The basic structure of the optical stripline is shown in Fig. 1. A similar structure, with region 4 replaced by a conducting plane and $d_2=0$, has been proposed for use at millimeter and submillimeter wavelengths [2]. Current research is aimed at producing circuit elements and system applications using these waveguides.

An overview of dielectric waveguides for microwave integrated circuits has been given by Knox [3]. Among the most recent applications are scannable antennas and tunable filters. Structures have been fabricated for use as electronic phase shifters at millimeter [4] and submillimeter wavelengths [5]. Itoh and Hebert [6] have simulated an electronically scannable antenna structure with a mechanical scan. A review of work in integrated optics has been given by Kogelnik [7]; many of the devices demonstrated use the optical stripline waveguide as a transmission element. An example is the directional coupler [8], which consists of several striplines spaced closely together to allow coupling between guides has been analyzed using coupled mode theories [9].

The effective index method [1] is a mathematically simple way of predicting some of the properties of optical striplines. This method explains light confinement under the stripe by assuming an effective index of refraction step in the lateral direction in region 3 of Fig. 1. This effective index step is found by first obtaining the propagation constants for the structure shown in Fig. 1 with $w=\infty$ and then with $w=0$. The difference between the two propagation constants divided by the free-space wavenumber gives the effective index step.

The far-field radiation pattern of a cleaved stripline radiating into free space is a useful way to characterize the structure. For example, the pattern can be measured experimentally and compared with these computed results to obtain values of the various waveguide parameters such as the refractive indices n_1, n_2, n_3, and n_4, or the dimensions d_2, d_3, and w. Another potential application of the stripline structure is in optical arrays where the waveguide is used as a member of the array. The pattern of the array of striplines is the product of the pattern of a single element and the array pattern.

In this paper we develop a series of rigorous calculations showing how the far-field radiation pattern behaves as a function of various waveguide parameters. In particular, we calculate the half-power beamwidth in the lateral direction of a waveguide mode radiating into free space. The value of the beamwidth is closely related to the effective lateral index step, and consequently determines the degree with which the waveguide mode is confined to the region below the stripe. For example, broad patterns are due to strongly confined modes. (These modes should be less vulnerable to losses introduced by lateral waveguide bends.)

A rigorous mathematical model of the optical stripline [10] does not require use of an effective lateral index step. An extension of this model has been developed for the structure in Fig. 1. The height is assumed to be infinite. Numerical results are given to show how the pattern

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depends on the parameters d_2, d_3, w, and the index discontinuity.

Since the effective index approximation involves considerably less computation, a method for obtaining information on the radiation pattern in the lateral direction using this approximation would be useful. One possible approach is to compute the effective index step in the lateral direction and then compute the radiation pattern of a symmetric three-layer waveguide with this index step and a thickness equal to the stripe width. We have used this approximate technique to compute the half-power beamwidth of the fundamental mode in the lateral direction. A comparison is given with the results obtained by using the more exact model presented in this paper.

II. Theory

The analysis of the waveguide structure will parallel that given in [10]. We will assume, following Marcatilli [11], that there are two sets of modes, one polarized along the x direction and one along the y direction. For simplicity, we will deal with the modes polarized along the x direction. We will further restrict this discussion to include only the even modes. The general solutions to the wave equation in the various regions can be written

$$\psi_1(x,y)=\sum_{n=1}^{\infty} A_n \cos p_n' x \exp\left(-p_n''(y-d_2)\right) \tag{1a}$$

$$\psi_2(x,y)=\int_0^{\infty}\cos qx\left[E(q)\exp\left(s''(y-d_2)\right) + F(q)\exp\left(-s''(y-d_2)\right)\right]dq \tag{1b}$$

$$\psi_3(x,y)=\int_0^{\infty}\cos qx\left[B(q)\sin q''y + C(q)\cos q''y\right]dq \tag{1c}$$

$$\psi_4(x,y)=\int_0^{\infty}\cos qx\left[D(q)\exp r''(d_3+y)\right]dq \tag{1d}$$

where

- ψ_i solution of the wave equation in the ith region,
- k_0 free-space wavenumber,
- n_i index of refraction in the ith region,
- k_i $=k_0 n_i$ (2)
- p_n' $=n\pi/2w$ (3)
- β propagation constant

$$p_n'^2-p_n''^2=k_1^2-\beta^2 \tag{4a}$$

$$q^2-s''^2=k_2^2-\beta^2 \tag{4b}$$

$$q^2+q''^2=k_3^2-\beta^2 \tag{4c}$$

$$q^2-r''^2=k_4^2-\beta^2. \tag{4d}$$

Matching the fields and their derivatives at the boundaries yields a secular equation of the form

$$\det(E-\Gamma)=0 \tag{5}$$

where E is the unit matrix and Γ is a matrix whose elements depend on the unknown propagation constant β. Since the matrices in (5) are of infinite order the eigenvalue solutions are found by taking an increasing matrix order beginning with a first-order matrix. This process forms a convergent sequence, converging to the true value of β. For the numerical results given in this paper, a fourth-order matrix was used. This order is sufficient since the sequence converges rapidly.

The constants B, C, D, E, and F of (1) are found by matching the fields or derivatives at the boundaries. The field description can then be found by substituting these constants into (1), giving

$$\psi_1(x,y)=\sum_{n=1}^{\infty} A_n \cos p_n' x \exp\left(-p_n''(y-d_2)\right) \tag{6}$$

$$\psi_i(x,y)=\sum_{n=1}^{\infty} A_n I_i, \qquad i=2,3,4 \tag{7}$$

where I_i represents an integral expression.

The radiation pattern is related to the Fourier transform of the field at the aperture [12]. Denote this transform by

$$\bar{e}(k_x,k_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{E}_f(x,y)\exp\left(i(k_x x+k_y y)\right)dx\,dy \tag{8}$$

where $\bar{E}_f$ represents the transverse facet electric field, and

$$k_x=k_0\cos\phi\sin\theta \tag{9a}$$

$$k_y=k_0\sin\phi\sin\theta \tag{9b}$$

where θ denotes the angle from the z axis and ϕ is the angle from the x axis in the xy plane. The far-field pattern for the case of an aperture field polarized in the x direction can be written

$$\bar{E}(\theta,\phi)=C\left[\hat{\theta}e_x\cos\phi-\hat{\phi}e_x\sin\phi\cos\theta\right] \tag{10}$$

where C is a constant, and $\hat{\theta}$ and $\hat{\phi}$ are unit vectors. It is, therefore, necessary to obtain $e_x(k_x,k_y)$ which is the Fourier transform of the field distribution given previously. The result can be written

$$e_x(k_x,k_y)=\sum_{i=1}^{4}F(\psi_i) \tag{11}$$

where $F(\psi_i)$ denotes the Fourier transform of ψ_i.

III. Numerical Results

The propagation constant for the structure in Fig. 1 with $d_2=0$ has been previously calculated [10]. The effect of region 2 on the propagation constant is shown in Fig. 2. We have used normalized values of the parameters so that the plot will be applicable to a wide class of structures. The normalized parameters are

$$D_2=d_2k_0\left(n_3^2-n_4^2\right)^{1/2} \tag{12a}$$

$$D_3=d_3k_0\left(n_3^2-n_4^2\right)^{1/2} \tag{12b}$$

$$W=wk_0\left(n_3^2-n_4^2\right)^{1/2} \tag{12c}$$

$$B^2=\frac{\beta^2}{k_0^2\left(n_3^2-n_4^2\right)}-\frac{n_4^2}{n_3^2-n_4^2} \tag{12d}$$

$$\eta=\frac{n_3^2-n_1^2}{n_3^2-n_4^2} \tag{12e}$$

$$\eta'=\frac{n_3^2-n_2^2}{n_3^2-n_4^2}. \tag{12f}$$

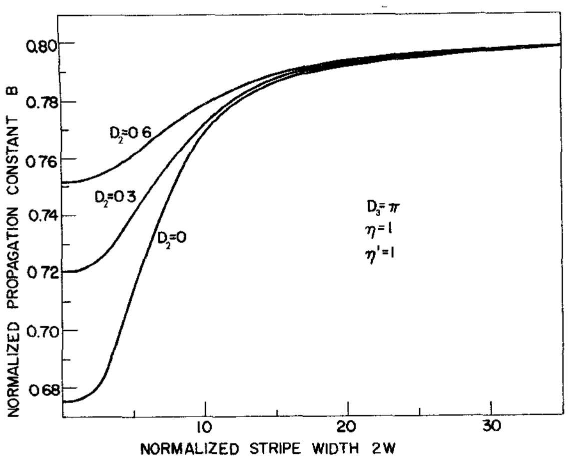

Fig. 2. Normalized propagation constant of the fundamental mode as a function of normalized stripe width.

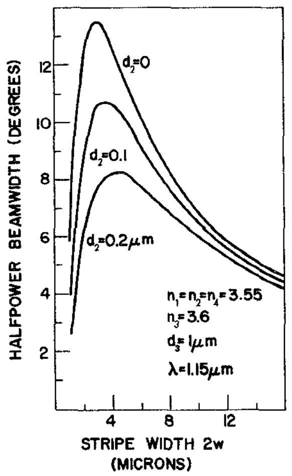

Fig. 3. Lateral half-power beamwidth of the fundamental mode as a function of stripe width.

Fig. 3 shows the normalized propagation constant as a function of $2W$, the normalized stripe width, for various values of D_2. The propagation constant approaches that of a three-layer waveguide for sufficiently large values of the stripe width. For smaller values of the normalized stripe width, which are more typical of the values used in integrated optics, the propagation constant is a strong function of D_2. For large enough values of D_2, we would simply obtain a three-layer waveguide, and the effect of the stripe would be inconsequential.

We now turn our attention to the radiation fields. The pattern of a waveguide radiating into free space depends on the actual values of the various waveguide parameters and thus cannot be expressed in terms of normalized parameters. Consequently, we chose values of the various parameters typical of those used in integrated optics. Consider a GaAs–AlGaAs waveguide with

$$n_1 = n_2 = n_4 = 3.55$$
$$n_3 = 3.6$$
$$d_3 = 1.0\ \mu\text{m}.$$

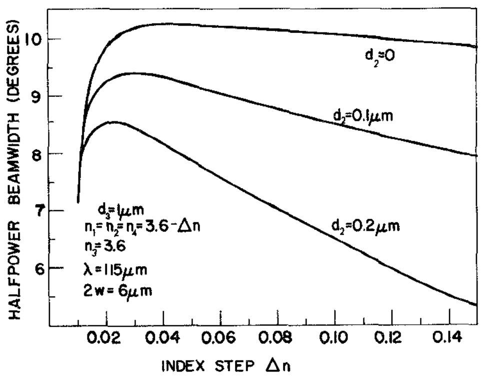

Fig. 4. Lateral half-power beamwidth of the fundamental mode as a function of the index discontinuity.

We have chosen a wavelength of 1.15 μm corresponding to one of the transitions in a HeNe laser. At $\lambda = 1.15$ μm absorption in GaAs is low.

The half-power beamwidth of the fundamental mode in the lateral direction is plotted as a function of the stripe width in Fig. 3. A scan in the lateral direction was chosen because the far-field beamwidth will indicate how well the fields are confined laterally. This is significant because confinement of the fields in the lateral direction does not occur because of an index discontinuity, and so it is important to understand the waveguide parameters which yield the greatest lateral optical confinement. The degree of optical confinement is important in considerations of scattering at waveguide bends.

From Fig. 3, it is apparent that the beamwidth decreases as d_2 increases. For small values of the stripe width, the beamwidth is a strong function of d_2. Each plot exhibits a maximum in the beamwidth which would correspond to an optimum value of w. Input coupling to the waveguide, which depends on the numerical aperture, would be maximized for this value of w. For $d_2 = 0$, for example, this optimum value would be a stripe width of approximately 3 μm.

In obtaining Fig. 4, a structure was used with $n_1 = n_2 = n_4, n_3 = 3.6$, $d = 1$ μm, $\lambda = 1.15$ μm, $w = 3$ μm, and the difference $n_3 - n_1$ was denoted Δn. This plot shows that the dependence of the lateral beamwidth on the transverse Δn is enhanced by the presence of region 4. Since a large Δn is desired for confinement in the vertical direction, this is the region of the plot which is of interest. Note that when $\Delta n \gtrsim 0.1$, the effect of the thickness of region 4 increasing from 0 to 0.2 μm is to decrease the half-power beamwidth by about 35 percent.

Fig. 5 shows the dependence of the lateral beamwidth on the thickness of the waveguide region 2. The structure has $n_1 = n_2 = n_4 = 3.55$, $n_3 = 3.6$, $\lambda = 1.15$, and $w = 3$ μm.

It is of interest to note that the effective index step approximation discussed earlier yields good results for the lateral half-power beamwidth. Fig. 6 shows plots of half-power beamwidth versus d_3 with $d_2 = 0$ computed using the theory above (curve 1) and using the effective index approximation (curve 2). It is apparent that the approxi-

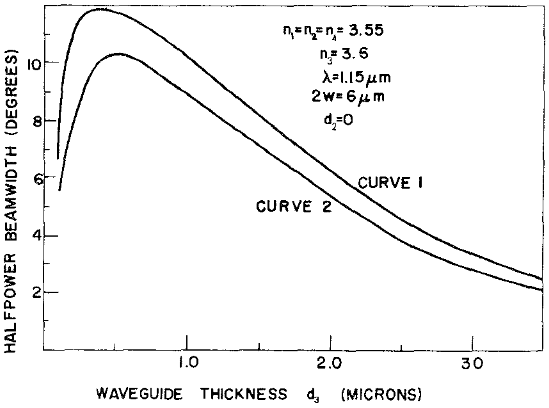

Fig. 5. Lateral half-power beamwidth of the fundamental mode versus the thickness of region 3, the region where light is confined.

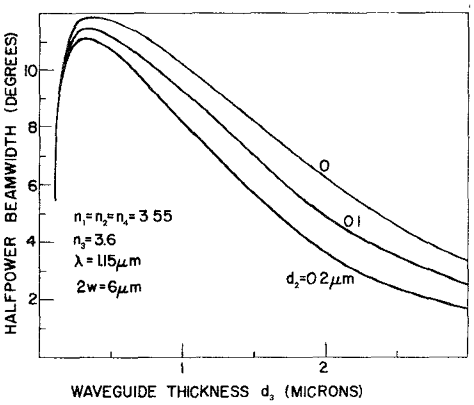

Fig. 6. Comparison of the method given in this paper for calculating the lateral half-power beamwidth (curve 1) with the effective index method (curve 2).

mation predicts the correct shape of the curve. The error which can be made using the approximation is less than 26 percent. If the pattern is needed only in the lateral direction, the use of the approximation may be justified by the savings in computation time.

IV. Conclusion

The propagation constant β of the fundamental mode in the optical stripline waveguide has been calculated. For small stripe widths the value of β is strongly dependent on the presence of a region between the stripe and the waveguiding region. As the thickness of the isolating layer increases, the effect of the strip width on β decreases.

The half-power beamwidth of the fundamental mode of the optical stripline waveguide in a plane parallel to the waveguide layers has been obtained; this is a useful indicator of the degree of optical confinement in the lateral direction. As the stripe widens the beamwidth decreases, indicating that the fields are spreading in the waveguide. As the stripe width is decreased beyond a critical point, the beamwidth begins to decrease; this indicates that the optical fields are no longer confined under the stripe. The effect of increasing the thickness of region 2 is always to reduce the half-power beamwidth.

The lateral beamwidth also depends on the thickness of the waveguiding region d_3. As d_3 is decreased the beamwidth increases, up to the point where d_3 becomes so small that the fields begin to spread outside the waveguiding region. Then the beamwidth drops sharply. This behavior is also predicted by using the effective index approximation to determine an index discontinuity, and then calculating the far-field beamwidth of a three-layer waveguide with this index step and a thickness of $2w$.

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